حل ورقة عمل الدرس الثاني

مشتقة الضرب والقسمة

(1) $f(x) = \dfrac{\pi}{(g(x))^2}$ $\qquad f'(2) = 4 \; , \; f'(2) = \pi$

$$f(x) = \pi\,(g(x))^{-2}$$

$$f'(x) = -2\pi\,(g(x))^{-3} * g'(x)$$

$$f'(2) = -2\pi\,(g(2))^{-3} * \underbrace{g'(2)}_{=4}$$

$$-\pi = -8\pi\,(g(2))^{-3}$$

$$= \frac{-8\pi}{g(2)^3} = -\pi$$

$$= \cancel{-\pi}\, g(2)^{+3} = \cancel{-}8\cancel{\pi}$$

$$g(2)^3 = 8$$

$$\boxed{g(2) = 2}$$

الجواب (a)

(2) إذا كان $f(x)=\sqrt[3]{x-2}$ فإن $f'(2)$:

a) صفر   b) 2   c) 1   (d) غير موجودة

$f(x)=(x-2)^{\frac{1}{3}}$

$f'(x)=\frac{1}{3}(x-2)^{-\frac{2}{3}}(1)$

$f'(x)=\frac{1}{3\sqrt[3]{(x-2)^2}}$

$f'(2)=\frac{1}{0}$ غير موجودة

(3) إذا كان $f(x)=\frac{\pi}{(g(x))^2}$ وكان $f'(2)=-\pi$ ، $g'(2)=4$

فإن قيمة $g(2)$ تساوي :

a) 2   b) -2   c) 8   d) -8

نجهز $f(x)=\pi(g(x))^{-2}$

نشتق $f'(x)=-2\pi(g(x))^{-3}g'(x)$

نعوض $f'(2)=-2\pi(g(2))^{-3}g'(2)$

$-\pi=-2\pi(g(2))^{-3}(4) \Rightarrow$

$-\pi=-8\pi(g(2))^{-3}$

$-\pi=\frac{-8\pi}{(g(2))^3}$

$-\pi(g(2))^3=-8\pi$

$(g(2))^3=\frac{-8\pi}{-\pi}$

$(g(2))^3=8 \Rightarrow \boxed{g(2)=2}$

(4) إذا كان $f(1)=2$ ، $g(1)=-1$ ، $f'(1)=2$ ، $g'(1)=-6$

فإن قيمة $\left(\frac{f+g}{g}\right)'(1)$ تساوي :

a) -2   b) 10   c) -10   d) 2

نجهز $\left(\frac{f+g}{g}\right)'(1)=\left(\frac{f}{g}+\frac{g}{g}\right)'(1)=\left(\frac{f}{g}+1\right)'(1)$

$=\frac{g(1)f'(1)-f(1)g'(1)}{(g(1))^2}+0=\frac{-1(2)-2(-6)}{(-1)^2}=\frac{-2+12}{1}$

$=10$

(2)

2

5) إذا كان $f(x) = x^n$ وكانت $f'''(x) = 210x^{n-3}$ فإن قيمة n :
حيث n عدد طبيعي

a) 12 b) 10 c) 7 d) 5

$f(x) = x^n$

$f'(x) = nx^{n-1}$

$f''(x) = n(n-1)x^{n-2}$

$f'''(x) = n(n-1)(n-2)x^{n-3}$

$210x^{n-3} = n(n-1)(n-2)x^{n-3}$

$210 = n(n-1)(n-2)$ $\Rightarrow$

$n(n-1)(n-2) = 210$

بالتخمين : المطلوب ثلاثة أعداد متتالية حاصل ضربها 210

$\boxed{n=5}$ $5(4)(3) = 60$ ✗

$\boxed{n=7}$ $7(6)(5) = 210$ ✓

6) إذا كان $f(x) = \frac{1}{a}x^n$ وكانت $f^{(3)}(x) = 5x^2$ فإن قيمة a تساوي

a) −5 b) 5 c) 12 d) −12

$f'(x) = \frac{n}{a}x^{n-1}$

$f''(x) = \frac{n}{a}(n-1)x^{n-2}$

$f^{(3)}(x) = \frac{n}{a}(n-1)(n-2)x^{n-3}$

$5x^2 = \frac{n}{a}(n-1)(n-2)x^{n-3}$ $\Rightarrow$

الأس = الأس

$n-3 = 2$

$\boxed{n=5}$

المعامل = المعامل

$\frac{n}{a}(n-1)(n-2) = 5$

$\frac{5}{a}(4)(3) = 5$

$\cancel{5}a = \cancel{5}$ (12)

$a = 12$

7) إذا كان $f(x)=3\sin x-\sin^3 x$ فإن $f'(x)$ تساوي

a) $3\sin^2 x$ b) $3\cos^2 x$ c) $3\sin^3 x$ d) $3\cos^3 x$

نبرر: $f(x)=3\sin x-(\sin x)^3$

نشتق: $f'(x)=3\cos x-3(\sin x)^2\cos x$

$f'(x)=3\cos x(1-\sin^2 x)$

$f'(x)=3\cos x\cos^2 x$

$f'(x)=3\cos^3 x$ d

8) إذا كان $y=\left(\frac{1+\sin x}{\cos x}\right)^n$ ، $\cos x\neq 0$ فإن $\frac{dy}{dx}$ تساوي

a) $n\sec x$ b) $n\tan x$ c) $ny\sec x$ d) $ny\tan x$

نبرر: $y=\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)^n$

$y=(\sec x+\tan x)^n$

نشتق: $\frac{dy}{dx}=n(\sec x+\tan x)^{n-1}(\sec x\tan x+\sec^2 x)$

$=n(\sec x+\tan x)^{n-1}\sec x\underbrace{(\tan x+\sec x)}$

$=n\sec x(\sec x+\tan x)^n$

$=n\sec x\ y$

9) إذا كان $f(x)=3\tan x+\tan^3 x$ فإن $f'(x)$ تساوي

a) $3\sec^2 x$ b) $3\tan^2 x$ c) $3\sec^4 x$ d) $3\tan$

نبرر: $f(x)=3\tan x+(\tan x)^3$

نشتق: $f'(x)=3\sec^2 x+3(\tan x)^2\sec^2 x$

$f'(x)=3\sec^2 x(\underbrace{1+\tan^2 x})$

$f'(x)=3\sec^2 x\sec^2 x$

$f'(x)=3\sec^4 x$

10) رائع

إذا كان $y = \sec x + \tan x$ فإن y' تساوي

a) $\frac{1}{1-\sin x}$ b) $\frac{1}{1+\sin x}$ c) $\frac{1}{1-\cos x}$ d) $\frac{1}{1-\sec x}$

$y = \sec x + \tan x$

$y' = \sec x \tan x + \sec^2 x$

$y' = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} + \frac{1}{\cos^2 x}$

$y' = \frac{\sin x}{\cos^2 x} + \frac{1}{\cos^2 x}$

$y' = \frac{\sin x + 1}{\cos^2 x} \Rightarrow$

$y' = \frac{\sin x + 1}{1 - \sin^2 x}$

$y' = \frac{\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}(1-\sin x)}$

$y' = \frac{1}{1-\sin x}$

11) إذا كان $y = \sec^3 2x$ فإن $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}}$ يساوي

a) 48 b) 24 c) $48\sqrt{3}$ d) صفر

نجهز $y = (\sec 2x)^3$

نشتق $y' = 3(\sec 2x)^2 \, 2\sec 2x \tan 2x$

نعوض $\left.y'\right|_{x=\frac{\pi}{6}} = 3\left(\sec\frac{\pi}{3}\right)^2 \, 2\sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) \Rightarrow$

$\left.y'\right|_{x=\frac{\pi}{6}} = 3(2)^2(2)(2)\sqrt{3}$

$= 48\sqrt{3}$

ثابتة ويساوي 1

12) إذا كان $y = \frac{2 - \sin(\frac{\pi}{2})}{\cos x}$ فإن $\frac{dy}{dx}$ تساوي

a) صفر b) $\sec x \tan x$ c) $-\csc x \cot x$ d) $\sin x$

نجهز $y = \frac{2-1}{\cos x} = \frac{1}{\cos x}$

$y = \sec x$

نشتق $y' = \sec x \tan x$

13) $f(x) = \dfrac{x - \sqrt{x}}{\sqrt{x} - 1} =$

$f''(x) = ??$

تجنس

$$= \frac{\sqrt{x}\,\cancel{(\sqrt{x} - 1)}}{\cancel{(\sqrt{x} - 1)}} = \sqrt{x}$$

عامل مشترك

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$$

$$f''(x) = \frac{-1}{4} x^{-\frac{3}{2}}$$

$$f''(x) = \frac{-1}{4\sqrt[2]{x^3}}$$

$$f''(4) = \frac{-1}{4\sqrt[2]{4^3}} = \frac{-1}{4(8)} = \frac{-1}{32}$$

$\boxed{d}$

14) $(f\cdot g)(x) = x^2+3$ $\qquad f(1) = g(1) = 2$, $g'(1) = 3$

$f'(1) = f(1)\,g'(1) + f'(1)\,g(1) = 2x$

$f'(1) = 2(3) + f'(1)(2) = 2$

$f'(1) = 6 + 2f'(1) = 2$

$\quad -\cancel{6} \qquad\qquad -\cancel{6}$

$2f'(1) = -4 \rightarrow \boxed{f'(1) = -2}$ (b)

15) $f(x) = \sqrt{3+\sin x+\cos x}$ $\qquad h'(0) = -1$, $h(0) = 2$

$K(x) = f(x).h(x)$ $\qquad K'(0) = ??$

$K'(0) = f'(0)\,h(0) + f(0)\,h'(0)$ $\qquad f'(0) = \dfrac{\cos x - \sin x}{2\sqrt{3+\sin x+\cos x}}$

$K'(0) = \frac{1}{4}\cdot(2) + (2)(-1) = \frac{2}{4} + -2 =$ $\qquad = \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

$= \frac{2}{4} - 2 = \boxed{\frac{-3}{2}}$ (c)

$f(0) = \sqrt{3+0+1}$

$= 2$

17) $f(x) = ax^2 + bx + c$ $\qquad f''(1) = 4$

$f'(1) = -2$

$f(1) = 3$

$f(1) = a+b+c = 3$

$f'(1) = 2ax + b$

$f'(1) = 2a + b = -2$

$f''(1) = 2a = 4 \rightarrow \boxed{a=2}$

$\boxed{f(x) = 2x^2 - 6x + 7}$ (b)

$2a + b = -2$

$4 + b = -2$ $\quad \boxed{b = -6}$

$2 - 6 + c = 3$ $\quad \boxed{c = 7}$

18) $y = x \tan x$

$y' = x \sec^2 x + \tan x$

$y'' = x(2\sec x)(\sec x \tan x) + \sec^2 x + \sec^2 x$

$y'' = 2x \sec^2 x \tan x + 2\sec^2 x$

$y'' = 2\sec^2 x(x \tan x + 1)$

$= 2\sec^2 x(y + 1)$

19) $y = (\sec x + \tan x)^n$

$y' = n(\sec x + \tan x)^{n-1}(\sec x \tan x + \sec^2 x)$

$y' = n(\sec x + \tan x)^{n-1}(\sec x(\tan x + \sec x))$

$= n\sec x(\sec x + \tan x)^n = n\sec x(y)$ $\boxed{c}$

20) $y = x\sin x + \cos x$

$y' = x\cos x + \cancel{\sin x} - \cancel{\sin x} = x\cos x$

$y'' = -x\sin x + \cos x$

$y'' = 0 + \cos 0 = \boxed{1}$ $\boxed{b}$

~~21) $f(x) = \frac{1}{2}x - \sin x$~~

~~$f'(x) = \frac{1}{2} - \cos x = 0 \rightarrow \cos x = \frac{1}{2}$, $x = \frac{\pi}{3}$~~

$\boxed{a}$

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21) $y = \tan x + \frac{2}{x-1}$, $x=0$

$m = \sec^2 x - \frac{2}{(x-1)^2}$

$f'(0) = 1 - \frac{2}{1} = \boxed{-1} = m$

$y = mx + b$

$-2 = -1x + b = -1(0) + b$

$-2 = b$

$\boxed{b = -2}$

$\boxed{y = -x - 2}$

$y = \tan 0 + \frac{2}{0-1}$

$\boxed{y = -2}$

C

22) $f(x) = \frac{1x}{2} - \sin x \qquad x \in [0, \frac{\pi}{2}]$

$m = f'(x) = \frac{1}{2} - \cos x = 0$

العمودي على المماس

موازي محور (y) يعني المماس يوازي محور (x)

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

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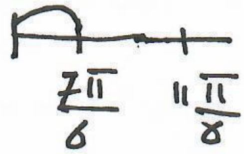
23) $S(t)=\frac{1}{2}t-\cos t$

* $v(t)=\frac{1}{2}+\sin t=0$ $\quad \sin t=-\frac{1}{2}$

عند أقصى ارتفاع

$t=\frac{7\pi}{6}, \; t=\frac{11\pi}{6}$



$$S\left(\frac{7\pi}{6}\right)=\frac{1}{2}\left(\frac{7\pi}{6}\right)-\cos\frac{7\pi}{6}$$
$$=\boxed{\frac{7\pi}{12}+\frac{\sqrt{3}}{2}}$$

* $v(t)=0$ عند أقصى إرتفاع

$S\left(\frac{7\pi}{6}\right)$ يكون الموقع

* وايضاً عندما يكون في حالة سكون لحظي يكون موقع الجسم

$$S\left(\frac{7\pi}{6}\right)=\frac{7\pi}{12}+\frac{\sqrt{3}}{2}$$

24) $f(x)=\frac{e^{3x}-1}{e^{x}-1}$

$f(x)=\frac{\cancel{(e^{x}-1)}(e^{2x}+2e^{x}+1)}{\cancel{e^{x}-1}}$ تجهيز السؤال

$f'(x)=2e^{2x}+2e^{x}$

$f''(x)=4e^{2x}+2$

$f'''(x)=8e^{2x}$

25) $y = \dfrac{x + 2\sqrt{x} + 1}{\sqrt{x} + 1}$

تجوّش السؤال

$y = \dfrac{(\sqrt{x} + 1)\cancel{(\sqrt{x} + 1)}}{\cancel{\sqrt{x} + 1}}$

$y' = \dfrac{1}{2\sqrt{x}} = 2x^{-\frac{1}{2}}$

$y'' = 2(-\frac{1}{2})(x)^{-\frac{2}{3}} = -x^{-\frac{2}{3}}$

$y''' = \frac{2}{3}(x)^{-\frac{5}{3}}$

$y'''' = y^{(4)} = \dfrac{-10}{9} x^{-\frac{8}{3}}$

$= \dfrac{-10}{9\sqrt[3]{x^8}}$

26) $f(x) = \sec x$

مماس أفقي

$[-2\pi, 2\pi]$

$f'(x) = \sec x \tan x = 0$

$= \dfrac{1}{\cos x} \dfrac{\sin x}{\cos x} = \dfrac{\sin x}{\cos^2 x} = 0$

$\sin x = 0 \rightarrow x =$

$-2\pi \quad -\frac{3\pi}{2} \quad -\frac{\pi}{2} \quad 0 \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad 2\pi$

$\boxed{x = 0, 2\pi, -2\pi}$

27) $g(x) = \dfrac{3x}{f(x)}$ $\qquad g'(0)$

$$g'(x) = \frac{(f(x))(3) - f'(x)(3x)}{(f(x))^2}$$

$$g'(0) = \frac{f(0)(3) - f'(0)(3(0))}{(f(0))^2}$$

من خلال الرسم

$$= \frac{(3)(3)}{(3)^2} = \boxed{1}$$

28) $f(x) = \sqrt{x-1}$

المماس موازي لمحور (y)

المقام = صفر

$$f'(x) = \frac{1}{2\sqrt{x-1}}$$

$x - 1 = 0$ $\quad \boxed{x = 1}$

29)

$$f(x) = \frac{x + \frac{b}{x}}{x+b} \qquad f'(1) = \frac{3}{2} \qquad b = ??$$

ثابتة

$$f(x) = \frac{\frac{x^2+b}{x}}{x+b} = \frac{x^2+b}{x(x+b)} = \frac{x^2+b}{x^2+bx}$$

$$f'(x) = \frac{[(x^2+bx)(2x)] - [(x^2+b)(2x+b)]}{(x^2+bx)^2}$$

$$= \frac{2x^3+2bx^2 - [2x^3+bx^2+2bx+b^2]}{(x^2+bx)^2}$$

$$= \frac{\cancel{2x^3}+2bx^2-\cancel{2x^3}-bx^2-2bx-b^2}{(x^2+bx)2}$$

$$= \frac{bx^2-2bx-b^2}{(x^2+bx)^2} = \frac{b(x^2-2x-b)}{(x^2+bx)^2}$$

$$f'(1) = \frac{b(1-2-b)}{(1+b)^2} = \frac{b(-1-b)}{(1+b)^2} = \frac{3}{2}$$

$$\frac{-b-b^2}{(1+b)^2} = \frac{3}{2} \Rightarrow 2(-b-b^2) = 3(1+b)^2$$

$$-2b-2b^2 = 3(1+b)^2$$

$$-2b-2b^2 = 3(1+2b+b^2)$$

$$-2b-2b^2 = 3+6b+3b^2$$

$$-2b-2b^2-3-6b-3b^2=0$$

$$-5b^2-8b-3=0$$

$$(b+1)(b+\frac{3}{5})=0$$

$$\boxed{b=-1 \qquad b=\frac{-3}{5}}$$

30) $f(x) = (2)^{g(x)}$ $\quad g'(1) = -2 \quad f'(1) = \ln \frac{1}{2}$

$g(1) = ??$

$f'(1) = 2^{g(1)} \cdot g'(1) \ln 2$

$\ln \frac{1}{2} = (2)^{g(1)} \cdot (-2) \ln 2$

$\ln(1) - \ln 2$

$$\frac{-\ln 2}{-2 \ln 2} = 2^{g(1)} \cdot \frac{(-2) \ln(2)}{-2 \ln 2}$$

$= \frac{1}{2} = 2^{g(1)}$

$\ln\left(\frac{1}{2}\right) = \ln(2)^{g(1)}$

$\ln\left(\frac{1}{2}\right) = g(1) \ln(2)$

$$\frac{-\ln 2}{\ln(2)} = g(1) \frac{\ln(2)}{\ln 2}$$

$\boxed{g(1) = -1}$